Stellar Structure and Asteroseismology

New Eyes to See Invisible Interiors of Stars

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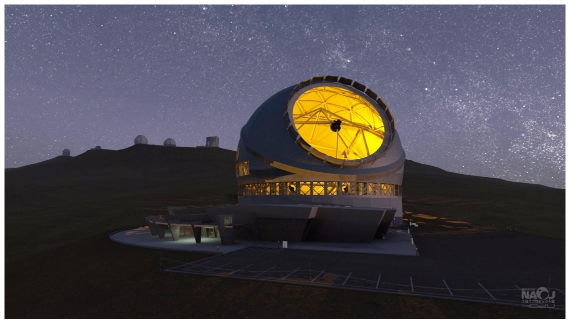
ALMA



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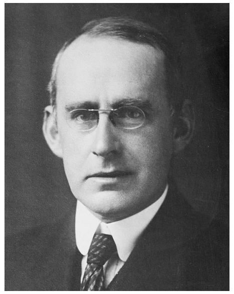
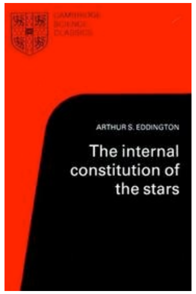




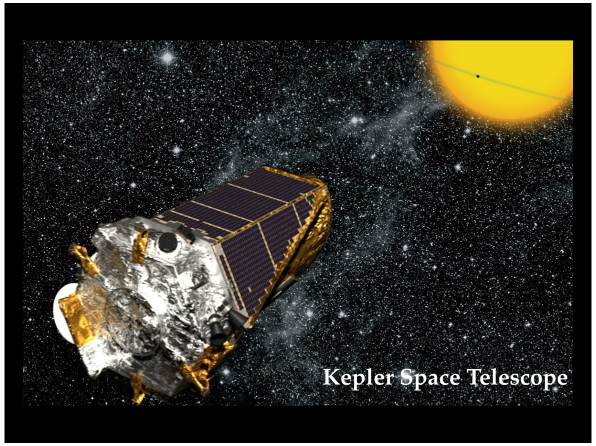
Arthur S. Eddington

At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?

- Arthur S. Eddington

Outlines

- Preparation: some basics
- Helioseismology: new eyes to see the invisible solar interior
- Some Topics of Asteroseismology
  - Internal Rotation of Stars
  - Finding Binaries through Asteroseismology
  - Super-Nyquist asteroseismology



Keplerian revolution

- Almost continuous observations over 4 years
- Observations from Space
  - no atmospheric scintillation
  - no day-night gaps
- Extremely high precision; $\Delta L/L \sim 10^{-6}$

I. Fundamentals of Stellar Physics

Observational Facts

Characteristic quantities

- Size — $R_{sun} = 7\ 10^8$ m
- Mass — $M_{sun} = 2\ 10^{30}$ kg
- Luminosity — $L_{sun} = 4\ 10^{26}$ W
- Dyn timescale — $\tau_{dyn} = (GM/R^3)^{-1/2} \sim 1$ hr
- Therm timescale — $\tau_{KH} = GM^2/(RL) \sim 10^7$ yr

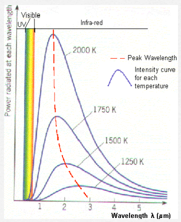
Light from a star

Thermal radiation from a body with T

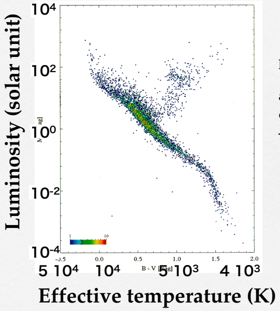
$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}$$

- Temperature determines spectrum
- Colour indicates temperature

$\lambda_{max}T$=2.9 10^{-3} m K



Effective Temperature vs Luminosity



parallax —> distance

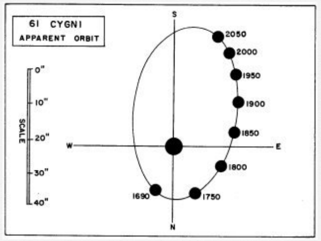
apparent brightness + distance
—> luminosity

Determination of Stellar Mass

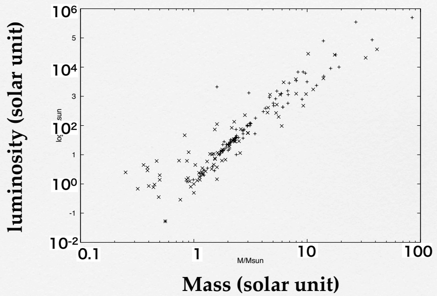
Observables：

(i) period P
(ii) parallax p
—> distance d
(iii) orbit size r_1&r_2

gravity = centrifugal force
$(r_1 + r_2)^3/P^2$
$= 4\pi^2 G(m_1+m_2)$
$r_1/r_2 = m_2/m_1$



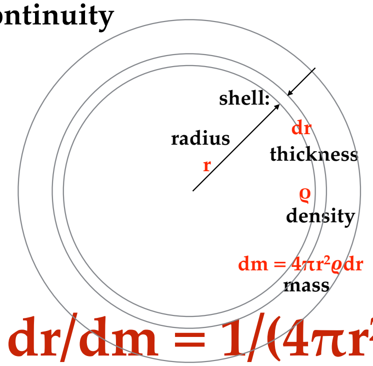
Mass-Luminosity relation



Theoretical consideration

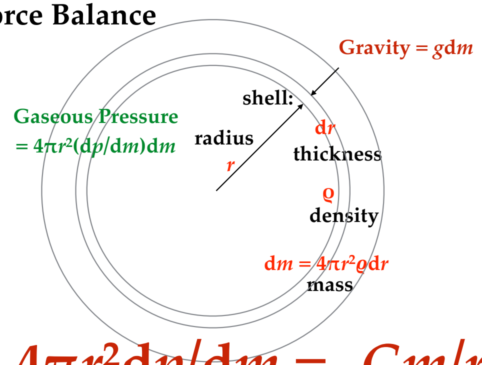
Why are stars shining?

Continuity



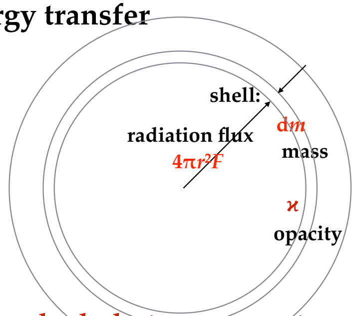
$$dr/dm = 1/(4\pi r^2 \rho)$$

Force Balance



$$4\pi r^2 dp/dm = -Gm/r^2$$

Energy transfer

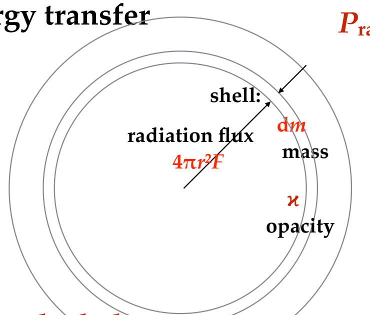


absorbed photon momentum = $F\varkappa \mathrm{d}m/c$

absorbed radiation energy = $F\varkappa \mathrm{d}m$

Energy transfer

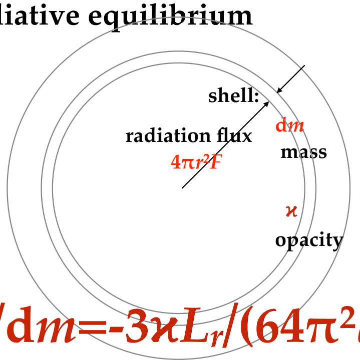
$P_{\mathrm{rad}} = aT^4/3$



absorbed photon momentum = $F\varkappa \mathrm{d}m/c$

radiation pressure = $-4\pi r^2(\mathrm{d}P_{\mathrm{rad}}/\mathrm{d}m)\mathrm{d}m$

Radiative equilibrium



$$\mathrm{d}T/\mathrm{d}m = -3\varkappa L_r/(64\pi^2 a c r^4 T^3)$$

Equilibrium state

$dr/dm = 1/(4\pi r^2 \varrho)$

$dp/dm = - Gm\varrho/(4\pi r^4)$

$dT/dm = - 3\kappa L_r/(64\pi^2 acr^4 T^3)$

Rough estimate:
Differential Eq. --> Difference Eq.
LHS: Difference between Surface and Center
RHS: Averaged values

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$dr/dm = 1/(4\pi r^2 \varrho)$

LHS $\approx R/M$
RHS $\approx (4\pi)^{-1}(R/2)^{-2}(\varrho_c/2)^{-1}$

$$\therefore \varrho_c \approx (2/\pi)(M/R^3)$$

Rough estimate:
Differential Eq. --> Difference Eq.
LHS: Difference between Surface and Center
RHS: Averaged values

$dp/dm = - Gm/(4\pi r^4)$

LHS $\approx -p_c/M$
RHS $\approx -G/(4\pi)\ (M/2)(R/2)^{-4}$

$$\therefore p_c \approx (2/\pi)(GM^2/R^4)$$

The central temperature

Ideal gas
$p = nkT$
$= (\varrho/\mu m_u)kT$

$\therefore T_c \approx (k/\mu m_u)^{-1} GM/R$

$\approx 10^7$ K for M_{sun} R_{sun}

n : particle numbers
μ : mean molecular weight
k = Boltzmann constant (1.38 10^{-23} J/K)
m_u = atomic weight (1.66 10^{-25} kg)

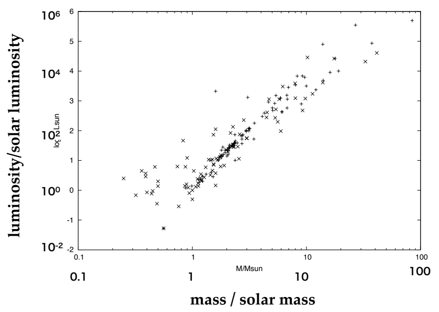
Rough estimate:
Differential Eq. --> Difference Eq.
LHS: Difference between Surface and Center
RHS: Averaged values

$dT/dm = - 3\kappa L_r/(64\pi^2 acT^3 r^4)$

LHS $\approx -T_c/M$
RHS $\approx -3\langle\kappa\rangle (L/2)/(64\pi^2 ac) (T_c/2)^{-3} (R/2)^{-4}$

$\therefore L \approx \pi^2/(3\langle\kappa\rangle) \{acG^4/(k/m_u)^4\} \mu^{-4} M^3$

Mass-Luminosity relation



Radiation from a Star

Stefan-Boltzmann law:
Radiation energy flux is proportional to T^4

$$L = A\int B_\lambda d\lambda = A\,\sigma T_{eff}^4$$

A = surface area (m^2) = $4\pi R^2$ (R: stellar radius)

Main Sequence

$L \approx \pi^2/(3\langle\kappa\rangle)\,\{acG^4/(k/m_u)^4\}\mu^{-4}M^3$

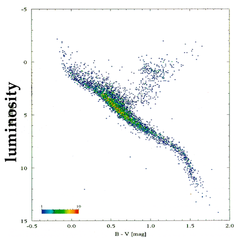
Normalizing with the solar values,

$\langle\kappa\rangle L/(\langle\kappa\rangle L)_{sun} \approx (M/M_{sun})^3$

Since $\sigma T_{eff}^4 = L/(4\pi R^2)$,

$$\begin{aligned}(T_{eff}/T_{eff,sun})^4 &= (L/L_{sun})(R/R_{sun})^{-2}\\ &\approx (L/L_{sun})(M/M_{sun})^{-2}\\ &\approx (L/L_{sun})^{1/3}\end{aligned}$$

$\therefore L/L_{sun} \propto (T_{eff}/T_{eff,sun})^{12}$



← surface temperature

Why are stars shining?

Nuclear fusion?

No!

Why are stars shining?

- Self gravity is supported by pressure.
- High gaseous pressure needs high temperature.
- Central temperature reaches 10^7 K.
- Energy flows from hot to cool regions.

Why are stars shining?

Simply because stars are hot !

Energy flows from hot to cool region.

Stellar Evolution

Star: Energy losing system

Energy loss = Cooling

Cooling timescale = $\int c_V T dm / L$

$\approx 10^7$ yr for the Sun !

Lifetime $\propto M/L \propto M^{-2}$

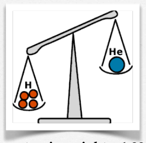
Necessity for sustaining mechanism

Nuclear fusion?

That's it!

Why can stars shine so long ?

mass = energy



H	atomic weight	1.008
He	atomic weight	4.002

{4m(H) - m(He)}/4 = 0.007

Assume
1. Solar composition: pure H
2. 10% of H converted to He

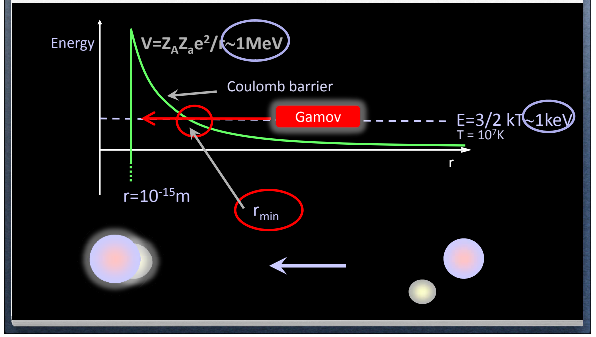
$E_{nuc} \simeq 0.007\ (0.1 M_{sun}) c^2$

$\simeq 1.3\ 10^{44}$ J

$t_{nuc} = E_{nuclear} / L_{sun}$

$\simeq 10^{10}$ yr

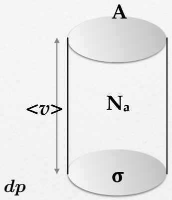
Coulomb barrier to nuclear fusion



Nuclear reactions

$r = N_A N_a \langle \sigma v \rangle$

Maxwell-Boltzmann distribution for v



$$\langle \sigma v \rangle = \int_0^\infty \sigma(v) v \frac{1}{(2\pi m_\mu kT)^{3/2}} e^{-E/kT} 4\pi p^2 \, dp$$

$$\boxed{\langle \sigma v \rangle = \int_0^\infty \frac{8\pi}{m_\mu^{1/2}} \frac{1}{(2\pi kT)^{3/2}} \sigma(E) E \, e^{-E/kT} \, dE}$$

Cross section

Geometrical factor $\sigma = \pi\lambda^2 \approx \pi h^2/(2m_\mu E)$

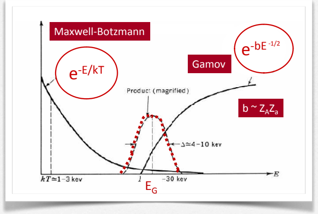
Penetration factor $P = \exp(-2\pi^2 r_{min}/\lambda)$
$\propto \exp(-\beta E^{-1/2})$

Nuclear factor $S(E)$

$$\boxed{\sigma(E) = E^{-1}\exp(-\beta E^{-1/2})\, S(E)}$$

Nuclear reactions

$$<\sigma v>= \left(\frac{8}{\pi m_\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \beta E^{-1/2}\right) dE$$

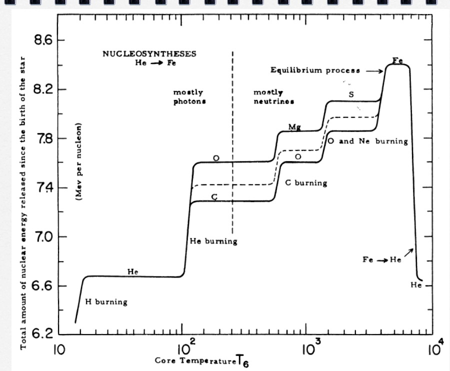


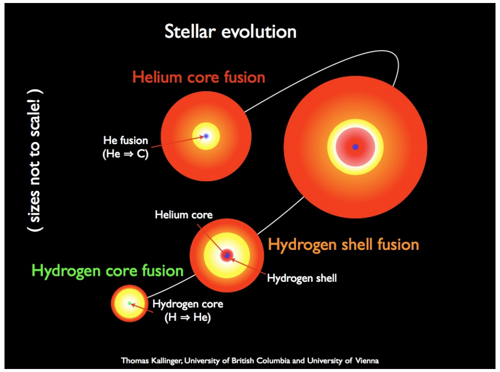
Gamow peak : $T \approx 10^7$ K
$\approx T_c$

Is this a coincidence?

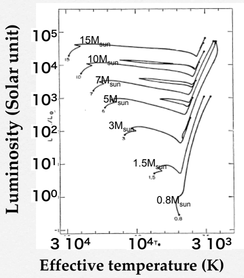
No! Stellar radius is adjusted so that

$R \approx (k/\mu m_u)^{-1} GM/T_c$





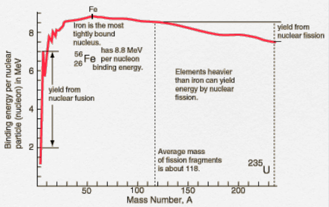
Stellar Evolution



- Mass-Luminosity relation $L \propto M^{\alpha}$
- $\tau \propto M/L \propto M^{1-\alpha}$
- Main Sequence -> Red Giants

Why Main Sequence?

Energy liberated by He and heavier nuclei is ~1/10 of the case of H burning



Hydrogen

Four hydrogen nuclei (4^1H) are converted to a helium nucleus (^4He)

atomic weight of H = 1.008, so 4.032 by 4^1H
atomic weight of He = 4.002

Hence, liberated energy per nucleus is proportional to (4.032 - 4.002)/4

Helium

Three helium nuclei (3^4He) are converted to a nucleus of carbon (^{12}C)

Atomic weight of He = 4.002, so 12.006 by 3^4He
Atomic weight of C = 12.000

Hence, liberated energy per nucleus is proportional to (12.006 - 12.000)/12

Lifetime of He burning will be shorter than that of H burning by a factor of
[(12.006 - 12.000)/12] / [(4.032 - 4.002)/4]

Essence of stellar evolution

- Toward gravitational contraction
  - However, its timescale is not GM^2/RL
- Residence by nuclear reactions
  - Timescales are governed by nuclear reactions

End of Lecture I